You have probably noticed that your grapher moves slowly when graphing NINT. This is because it must compute each value as a limit of sums—comparatively slow work even for a microprocessor. Here are some ways to speed up the process:

Change the *x-resolution.* The default resolution is 1, which means that the grapher will compute a function value for every vertical column of pixels. At resolution 2 it computes only every second value, and so on. With higher resolutions, some graph smoothness is sacrificed for speed.

Fundamental Theorem of Calculus

THEOREM 4 The Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a, b]$, then the function

$$F(x) = \int_a^x f(t)\,dt$$

has a derivative at every point x in $[a, b]$, and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x). \tag{1}$$

EXAMPLE 1 Applying the Fundamental Theorem

Find

$$\frac{d}{dx}\int_{-\pi}^{x} \cos t \, dt \quad \text{and} \quad \frac{d}{dx}\int_{0}^{x} \frac{1}{1+t^2}\, dt$$

by using the Fundamental Theorem.

EXAMPLE 2 The Fundamental Theorem with the Chain Rule

Find dy/dx if $y = \int_1^{x^2} \cos t \, dt$.

EXAMPLE 3 Variable Lower Limits of Integration

Find dy/dx.

(a) $y = \int_{x}^{5} 3t \sin t \, dt$ **(b)** $y = \int_{2x}^{x^2} \frac{1}{2 + e^t} \, dt$

Example 1

Given $\frac{dy}{dx} = 3x^2 + 4x - 5$ with the initial condition $y(2) = -1$, find $y(3)$.

Method 1:

Method 2:

EXAMPLE 4 Constructing a Function with a Given Derivative and Value

Find a function $y = f(x)$ with derivative

$$\frac{dy}{dx} = \tan x$$

that satisfies the condition $f(3) = 5$.

$$\int_3^x \tan t \, dt \quad + 5$$

EXPLORATION 2 The Effect of Changing a in $\int_a^x f(t)dt$

The first part of the Fundamental Theorem of Calculus asserts that the derivative of $\int_a^x f(t)\,dt$ is $f(x)$, regardless of the value of a.

1. Graph NDER (NINT (x^2, x, 0, x)).
2. Graph NDER (NINT (x^2, x, 5, x)).
3. Without graphing, tell what the x-intercept of NINT (x^2, x, 0, x) is. Explain.
4. Without graphing, tell what the x-intercept of NINT (x^2, x, 5, x) is. Explain.
5. How does changing a affect the graph of $y = (d/dx)\int_a^x f(t)\,dt$?
6. How does changing a affect the graph of $y = \int_a^x f(t)\,dt$?

THEOREM 4 (continued) **The Fundamental Theorem of Calculus, Part 2**

If f is continuous at every point of $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

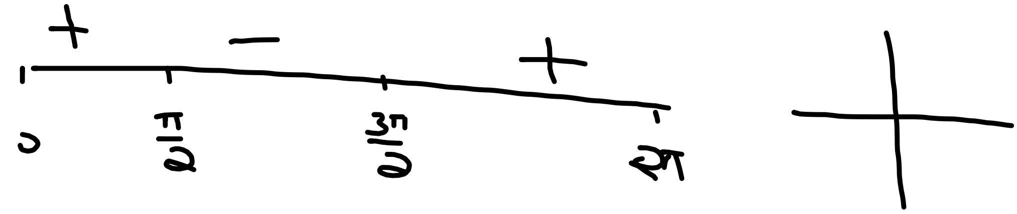
This part of the Fundamental Theorem is also called the **Integral Evaluation Theorem.**

$$H(x) = \int_{\frac{\pi}{2}}^{x} t \cos t \, dt \qquad 0 < x < 2\pi$$

a) $H'(x) = x\cos x$

$x\cos x = 0$

$\cancel{x=0}$, $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$



max at $x = \frac{\pi}{2}$ because $H'(x) > 0$ when $0 < x < \pi/2$ and $H'(x) < 0$ when $\pi/2 < x < 3\pi/2$

min at $x = \frac{3\pi}{2}$ because $H'(x) < 0$ when $\frac{\pi}{2} < x < \frac{3\pi}{2}$ and $H'(x) > 0$ when $\frac{3\pi}{2} < x < 2\pi$

$$F(x) = \int_1^{2x} f(t)\,dt$$

a) $$F(0) = \int_1^{2(0)} f(t)\,dt$$

$$= \int_1^{0} f(t)\,dt$$

$$= -\int_0^{1} f(t)\,dt = -1.3$$

$$F(1) = \int_1^{2} f(t)\,dt = -1.3$$

b) $$F'(x) = 2 \cdot f(2x)$$

c) $$2 \cdot f(2x) = 0$$

when $f(2x) = 0$

$2x = 1 \qquad 2x = 4$

$x = \frac{1}{2} \qquad x = 2$

d)

$$\begin{array}{ccccccc} & + & & - & & + & \\ 0 & & \frac{1}{2} & & 2 & & 3 \end{array}$$

max at $x = 1/2$ . . . .

min at $x = 2$ . . . .

EXAMPLE 5 Evaluating an Integral

Evaluate $\int_{-1}^{3}(x^3 + 1)\,dx$ using an antiderivative.

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\frac{1}{4}x^4 + x \Big|_{-1}^{3}$$

$$= \left(\frac{1}{4}(3)^4 + 3\right) - \left(\frac{1}{4}(-1)^4 + (-1)\right)$$

$$= \left(\frac{81}{4} + 3\right) - \left(\frac{1}{4} - 1\right)$$

$$= \left(\frac{81}{4} + \frac{12}{4}\right) - \left(\frac{1}{4} - \frac{4}{4}\right)$$

$$\frac{93}{4} + \frac{3}{4} = \frac{96}{4}$$

$$= 24$$

initial condition

Example 5

A pizza with a temperature of 95°C is put into a 25°C room when $t = 0$. The pizza's temperature is decreasing at a rate of $r(t) = 6e^{-0.1t}$ °C per minute. Estimate the pizza's temperature when $t = 5$ minutes.

when $t = 0$

$H(t) = 95$

$H(t)$ (temp function)

$$\underbrace{\int_0^5 -6e^{-0.1t}\,dt}_{\text{(change in temp from 0 to 5)}} + 95$$

≈ 71.392°C

Total Area vs Net Area

EXAMPLE 6 Finding Area Using Antiderivatives

Find the area of the region between the curve $y = 4 - x^2$, $0 \le x \le 3$, and the x-axis.

How to Find Total Area Analytically

To find the area between the graph of $y = f(x)$ and the x-axis over the interval $[a, b]$ analytically,

1. partition $[a, b]$ with the zeros of f,
2. integrate f over each subinterval,
3. add the absolute values of the integrals.

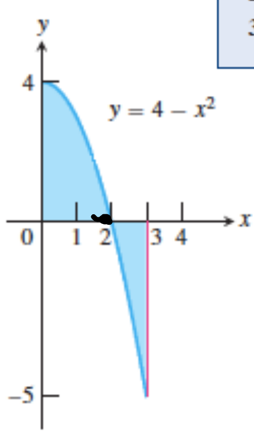


Figure 5.28 The function $f(x) = 4 - x^2$ changes sign only at $x = 2$ on the interval $[0, 3]$. (Example 6)

$$\int_0^2 (4 - x^2)\,dx + \left|\int_2^3 (4 - x^2)\,dx\right|$$

$$4x - \frac{1}{3}x^3 \Big|_0^2 + \left| 4x - \frac{1}{3}x^3 \Big|_2^3 \right|$$

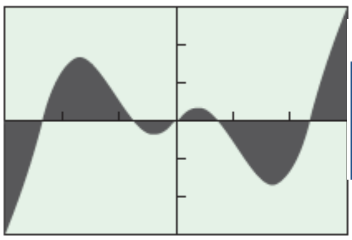
$$\left(8 - \frac{8}{3}\right) - 0 \qquad \left|(12 - 9) - \left(8 - \frac{8}{3}\right)\right|$$

$$\left|3 - \frac{16}{3}\right|$$

$$\frac{16}{3} + \left|-\frac{7}{3}\right|$$

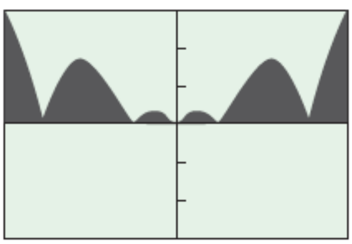
$$= \frac{23}{3}$$

We can find area numerically by using NINT to integrate the *absolute value* of the function over the given interval. There is no need to partition. By taking absolute values, we automatically reflect the negative portions of the graph across the x-axis to count all area as positive (Figure 5.29).



[–3, 3] by [–3, 3]

(a)



[–3, 3] by [–3, 3]

(b)

Figure 5.29 The graphs of (a) $y = x\cos 2x$ and (b) $y = |x\cos 2x|$ over $[-3, 3]$. The shaded regions have the same area.

EXAMPLE 7 Finding Area Using NINT

Find the area of the region between the curve $y = x\cos 2x$ and the x-axis over the interval $-3 \le x \le 3$ (Figure 5.29).

How to Find Total Area Numerically

To find the area between the graph of $y = f(x)$ and the x-axis over the interval $[a, b]$ numerically, evaluate

$$\text{NINT}\,(|f(x)|, x, a, b).$$

EXAMPLE 8 Using the Graph of f to Analyze $h(x) = \int_a^x f(t)\,dt$

The graph of a continuous function f with domain [0, 8] is shown in Figure 5.30. Let h be the function defined by $h(x) = \int_1^x f(t)\,dt$.

(a) Find $h(1)$. $\int_1^1 f(t)\,dt = 0$

(b) Is $h(0)$ positive or negative? Justify your answer. negative

$h'(x) = f(x)$

(c) Find the value of x for which $h(x)$ is a maximum. $x = 4$

(d) Find the value of x for which $h(x)$ is a minimum. $x = 0$, $x = 8$

(e) Find the x-coordinates of all points of inflections of the graph of $y = h(x)$.

$x = 1$, $x = 3$, $x = 6$

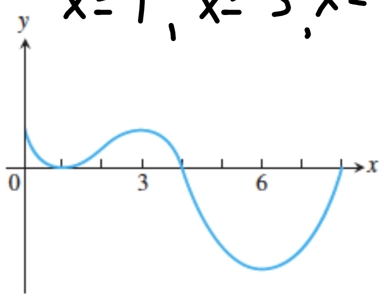


Figure 5.30 The graph of f in Example 8, in which questions are asked about the function $h(x) = \int_1^x f(t)\,dt$.

because

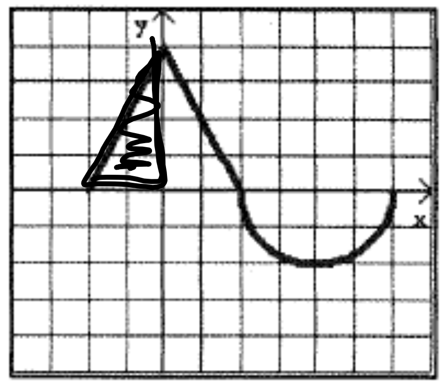
$\int_0^1 f(t)\,dt$ is positive

so $\int_1^0 f(t)\,dt$ would be negative

Example 3

The graph of f' on $-2 \le x \le 6$ consists of two line segments and a semicircle as shown at right. Given that $f(-2) = 5$, initial condition

find $f(0)$, $f(2)$, and $f(6)$.



Graph of f'

$$f(0) = \int_{-2}^{0} f'(x)\,dx + 5$$

$$= 4 + 5 = 9$$

$f(2) =$